MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2022/2023 (T2230)

EMT2066 – INDUSTRIAL MATHEMATICS

(All Sections / Groups)

3rd OCTOBER 2023

9.00 AM – 11.00 AM

(2 Hours)

INSTRUCTIONS TO STUDENTS:

1. This exam paper consists of **8 pages** (including cover page) with **4 questions** only.
2. Attempt **all** 4 questions. All questions carry equal marks and the distribution of marks for each question is given.
3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.
4. Several formulae are provided in the **Appendix** for your reference.

Question 1 (Numerical Methods)

(a) Consider the integral $\int_1^2 \ln(x+1)\,dx$.

(i) Approximate the integral using composite trapezoidal rule with 6 intervals.

[5 marks]

(ii) Compute the error bound for the approximation in (a)(i).

[5 marks]

(b) Consider the following system of linear equations:

$$\begin{aligned} 5x_1 - 2x_2 + 3x_3 + 1 &= 0 \\ 2x_1 - x_2 - 7x_3 &= 3 \\ -3x_1 + 9x_2 + x_3 &= 2 \end{aligned}$$

By using Jacobi technique, estimate $[x_1, x_2, x_3]$ up to **two iterations** starting from an initial estimate $[x_1, x_2, x_3] = [0,0,0]$. (**Hint**: Is the system diagonally dominant?)

[15 marks]

Continued...

Question 2 (Stochastic Processes)

(a) A Markov chain with state space $S = \{1, 2, 3, 4\}$ is governed by the transition probability matrix

$$\mathbf{P} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 1/4 & 3/4 \end{bmatrix}.$$

(i) Draw the state transition diagram representing the chain.

[4 marks]

(ii) Decompose the state space into equivalence classes.

[2 marks]

(iii) Which states are transient and which are recurrent? Justify your answer.

[2 marks]

(iv) Determine the periods of state 1 and state 2. Are these states periodic? Why or why not?

[3 marks]

(v) Evaluate $P_{42}^{(4)}$, the probability of moving from state 4 to state 2 in exactly 4 transitions.

[4 marks]

(vi) Suppose that the Markov chain have initial state probabilities $\mathbf{p}(0) = [0 \quad 1/5 \quad 4/5 \quad 0]$. After 2 transitions, in which state will the chain most likely be found?

[3 marks]

(b) A binary digital transmitter sends data as a stream of 0's and 1's. From the bit pattern, it is observed that bit 0 is followed by bit 1 with probability 0.75. On the other hand, bit 1 is followed by bit 0 with probability 0.5. Construct the transition probability matrix that models the output of the transmitter. Hence, determine the proportion of bit 1's in the data stream after an extended period of time.

[7 marks]

Continued...

Question 3 (Operations Research)

(a) Consider the following linear programming (LP) problem:

$$\begin{aligned} \text{Maximize } z = \; & 5x_1 + 10x_2 + 8x_3 \\ \text{subject to:} \quad & 3x_1 + 5x_2 + 2x_3 \le 60 \quad \text{(Constraint 1)} \\ & 4x_1 + 4x_2 + 4x_3 \le 72 \quad \text{(Constraint 2)} \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

By introducing slack variables where appropriate, express the LP problem in **standard form**. Then solve the LP problem using the **simplex method**. Be sure to specify the **entering and leaving variables**, as well as the **elementary row operations** for each iteration. For the benefit of comparison, a partially filled simplex tableau at the optimal stage is shown below:

Basic	z	x_1	x_2	x_3	s_1	s_2	Solution
z	1	$\frac{11}{3}$	0	0	$\frac{2}{3}$	$\frac{5}{3}$	160
?	0	?	?	?	?	?	8
?	0	?	?	?	?	?	10

[15 marks]

(b) Convert the above LP problem to its **dual**.

[5 marks]

(c) Using appropriate properties, deduce the optimal solution and optimal value for the dual problem in (b).

[3 marks]

(d) What would happen to the optimal value if:

(i) the right-hand-side (RHS) of Constraint 1 is increased by 1 unit?

(ii) the RHS of Constraint 2 is decreased by 1 unit?

[2 marks]

Continued...

Question 4 (Operations Research)

(a) David lives in Town A. He wishes to visit his friend who stays in Town F. **Figure Q4a** below shows possible driving routes that connect Town A to Town F. David has to make 2 pit stops before reaching Town F. At the first stop, he can choose between Town B and Town C. At the second stop, he can choose between Town D and Town E.

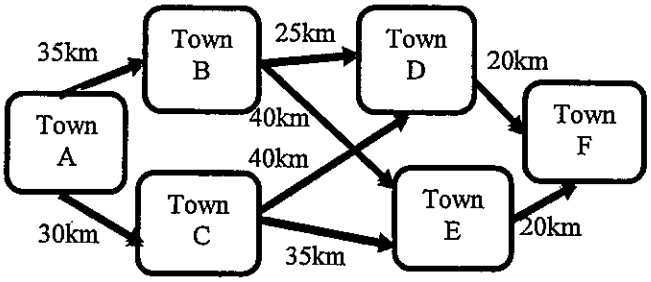

Figure Q4a

Using Dynamic Programming (DP), determine the route from Town A to Town F that minimizes David's driving distance. In your solution, you must:

1. Define clearly what the stages, states, recursive formulae and decision variables of your DP model represent.
2. Indicate the applicable states for each stage and show the necessary computations for the recursive formulae in obtaining the decision variable.

Finally, state the optimal policy (route) and the corresponding optimal distance.

[13 marks]

(b) Using Kruskal's algorithm, demonstrate how the network in **Figure Q4b** may be reduced to a minimum spanning tree.

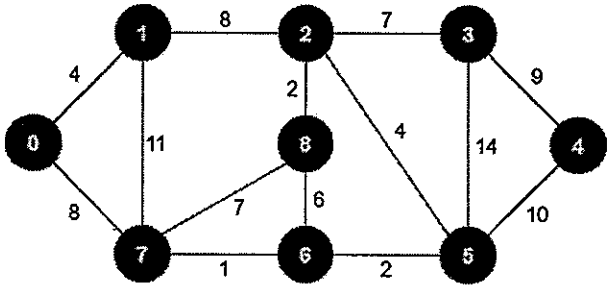

Figure Q4b

[12 marks]

Continued...

APPENDIX: Selected Formulae for Numerical Methods

1. **Lagrange interpolating polynomial (LIP)**

$$P_n(x)=\sum_{i=0}^{n} L_i(x)f(x_i)$$

where $L_k(x)=\prod_{\substack{i=0\\ i\neq k}}^{n}\frac{(x-x_i)}{(x_k-x_i)}$.

2. **Newton's divided-difference interpolating polynomial (NDDIP)**

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} f[x_0, x_1, ..., x_k](x-x_0)\cdots(x-x_{k-1}),$$

3. **Newton's forward-difference interpolating polynomial**

$$P_n(x) = f[x_0] + \sum_{k=1}^{n}\binom{s}{k}\Delta^k f(x_0).$$

4. **Newton's backward-difference interpolating polynomial**

$$P_n(x) = f[x_n] + \sum_{k=1}^{n}(-1)^k\binom{-s}{k}\nabla^k f(x_n).$$

5. **Error in interpolating polynomial**

$$f(x)-P_n(x)=\frac{(x-x_0)(x-x_1)...(x-x_n)}{(n+1)!}f^{(n+1)}(c_x).$$

6. **Forward difference formula for differentiation**

$$f'(x)\approx\frac{f(x+h)-f(x)}{h}.$$

7. **Backward difference formula for differentiation**

$$f'(x)\approx\frac{f(x)-f(x-h)}{h},$$

where the error bound for both forward and backward difference is $\left|\frac{h}{2}f''(c_x)\right|$.

9. **Central difference formula for differentiation**

$$f'(x)\approx\frac{f(x+h)-f(x-h)}{2h}$$

with error bound $\left|\frac{h^2}{6}f^{(3)}(c_x)\right|$.

Continued...

10. **Trapezoidal rule**

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\big(f(a)+f(b)\big),$$

where $h = b - a$ and the error bound is $\left|\frac{h^3 f^{"}(\xi)}{12}\right|$.

11. **Composite Trapezoidal rule**

$$\int_a^b f(x)dx \approx \frac{h}{2}\left[f(a)+f(b)+2\sum_{j=1}^{n-1} f(x_j)\right],$$

where $h = \frac{b-a}{n}$, $x_j = a + jh$ and the error bound is $\left|\frac{(b-a)h^2 f''(\xi)}{12}\right|$.

12. **Simpson's rule**

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right],$$

where $h = \frac{b-a}{2}$ and the error bound is $\left|\frac{h^5}{90} f^{(iv)}(\xi)\right|$.

13. **Composite Simpson's rule**

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a)+2\sum_{j=1}^{\left(\frac{n}{2}\right)-1} f(x_{2j})+4\sum_{j=1}^{\frac{n}{2}} f(x_{2j-1})+f(b)\right],$$

where $h = \frac{b-a}{n}$, $x_j = a + jh$ and the error bound is $\left|\frac{(b-a)h^4}{180} f^{(4)}(\xi)\right|$.

14. **Gaussian Quadrature**

$$\int_a^b f(x)\,dx = \frac{(b-a)}{2}\int_{-1}^{1} f\left(\frac{(b+a)+t(b-a)}{2}\right)dt \approx w_1 f(t_1) + w_2 f(t_2) + \ldots + w_n f(t_n)$$

n	Nodes t_i	Weights, w_i
2	−0.57735 02692	1
	0.57735 02692	1
3	−0.77459 66692	0.5555 55556
	0	0.8888 88889
	0.77459 66692	0.5555 55556
4	−0.86113 63116	0.34785 48451
	−0.3399810436	0.65214 51549
	0.3399810436	0.65214 51549
	0.86113 63116	0.34785 48451

15. **Newton-Raphson's method**

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})},\ n = 1,2,...$$

15. **Euler's method**

$$y_{i+1} = y_i + hf(x_i, y_i)$$

16. **Runge Kutta method of order two** (Improved Euler method)

$$y_{i+1} = y_i + \tfrac{1}{2}(k_1 + k_2)$$
$$k_1 = hf(x_i, y_i)$$
$$k_2 = hf(x_i + h, y_i + k_1)$$

17. **Runge Kutta method of order four**

$$k_1 = hf(x_i, y_i),$$
$$k_2 = hf(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_1),$$
$$k_3 = hf(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_2),$$
$$k_4 = hf(x_{i+1}, y_i + k_3),$$
$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$

18. **Jacobi Method**

$$x_i^{(k)} = \frac{\sum_{\substack{j=1 \\ j \neq i}}^{n} \left(-a_{ij} x_j^{(k-1)}\right) + b_i}{a_{ii}},\ \text{for } i = 1,2,...,n.$$

19. **Gauss-Siedel Method**

$$x_i^{(k)} = \frac{-\sum_{j=1}^{i-1} \left(a_{ij} x_j^{(k)}\right) - \sum_{j=i+1}^{n} \left(a_{ij} x_j^{(k-1)}\right) + b_i}{a_{ii}},\ \text{for } i = 1,2,...,n.$$

End of Paper.